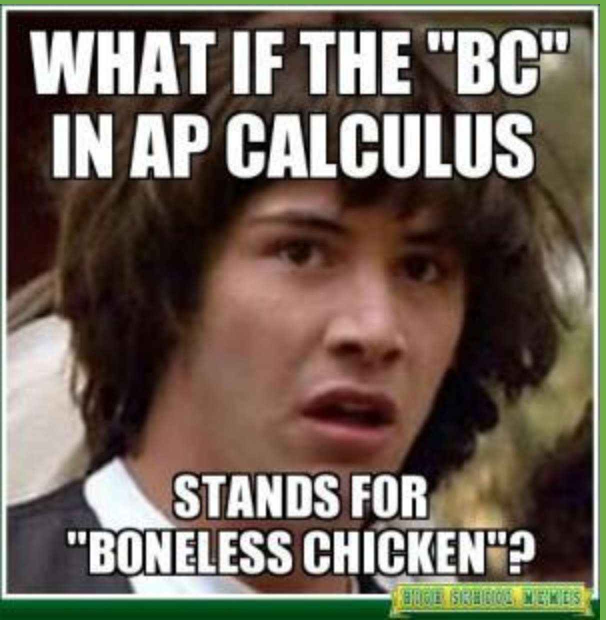
AP CALCULUS BC SAMPLE EXAM QUESTIONS



A calculator may not be used on questions on this part of the exam.

1. A curve is defined by the parametric equations $x(t) = 3e^{2t}$ and $y(t) = e^{3t} - 1$.

What is $\dfrac{d^2y}{dx^2}$ in terms of t?

(A) $\dfrac{1}{12e^t}$

(B) $\dfrac{1}{9e^t}$

(C) $\dfrac{e^t}{2}$

(D) $\dfrac{3e^t}{4}$

2.

$x_0 = 0$	$f(x_0) = 2$
$x_1 = 2$	$f(x_1) \approx 6$
$x_2 = 4$	$f(x_2) \approx 10$

Consider the differential equation $\frac{dy}{dx} = \frac{Ax^2 + 4}{y}$, where A is a constant.

Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 2$. Euler's method, starting at $x = 0$ with a step size of 2, is used to approximate $f(4)$. Steps from this approximation are shown in the table above. What is the value of A ?

(A) $\frac{1}{2}$

(B) 2

(C) 5

(D) $\frac{13}{2}$

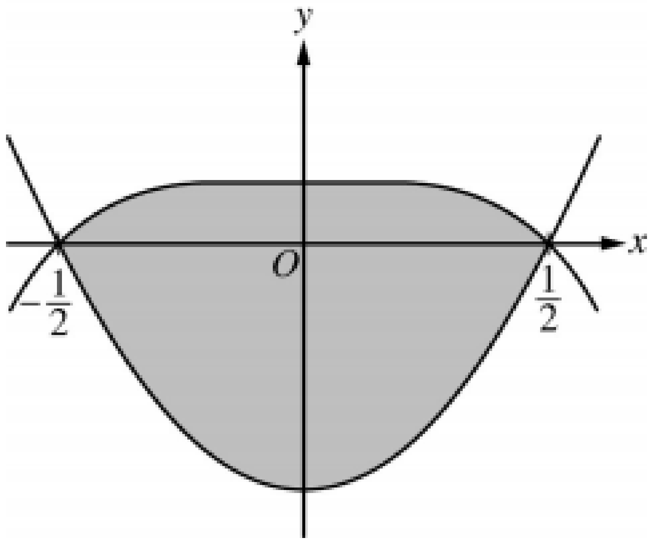
3. $\displaystyle\int \frac{12}{(x-1)(x-5)}\,dx =$

(A) $-3\ln|x-1| + 3\ln|x-5| + C$

(B) $-2\ln|x-1| + 2\ln|x-5| + C$

(C) $3\ln|x-1| - 3\ln|x-5| + C$

(D) $12\ln|x-1| + 12\ln|x-5| + C$



4. The shaded region in the figure above is bounded by the graphs of $y = x^2 - \frac{1}{4}$ and $y = \frac{1}{16} - x^4$ for $-\frac{1}{2} \le x \le \frac{1}{2}$. Which of the following expressions gives the perimeter of the region?

(A) $2\int_0^{1/2} \sqrt{4x^2 + 16x^6}\, dx$

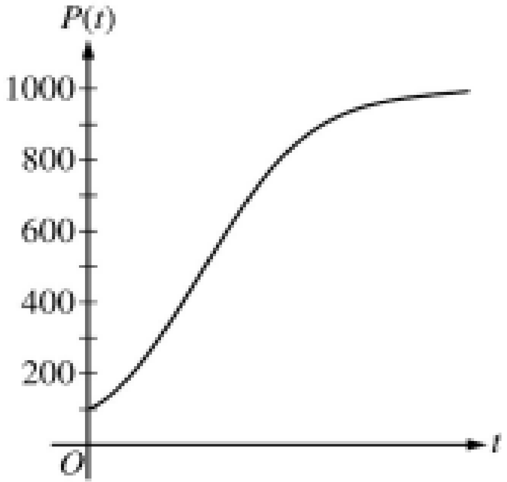
(B) $2\int_0^{1/2} \sqrt{1 + 4x^2 + 16x^6}\, dx$

(C) $2\int_0^{1/2} \sqrt{1 + 4x^2}\, dx + 2\int_0^{1/2} \sqrt{1 + 16x^6}\, dx$

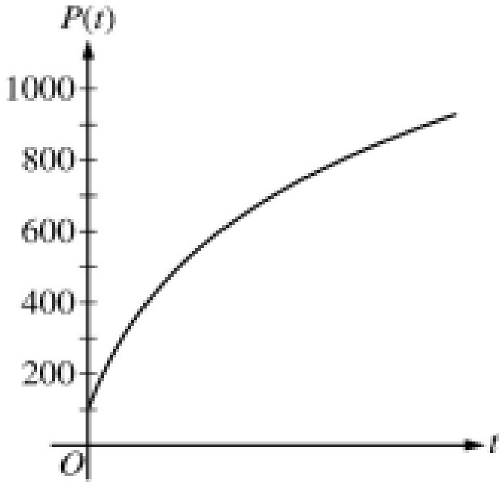
(D) $2\int_0^{1/2} \sqrt{1 + \left(x^2 - \frac{1}{4}\right)^2}\, dx + 2\int_0^{1/2} \sqrt{1 + \left(\frac{1}{16} - x^4\right)^2}\, dx$

5. The number of fish in a lake is modeled by the function P that satisfies the differential equation $\frac{dP}{dt} = 0.003P(1000 - P)$, where t is the time in years. Which of the following could be the graph of $y = P(t)$?

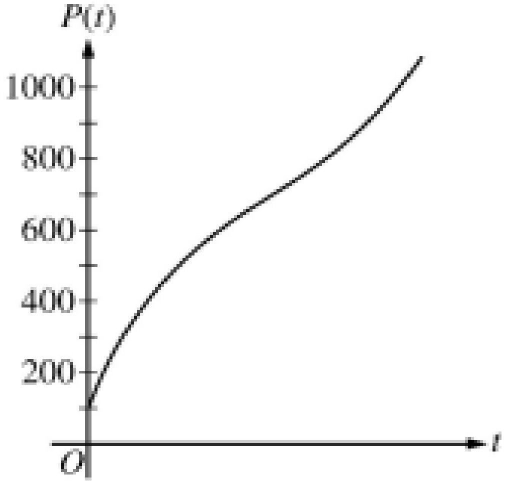
(A)



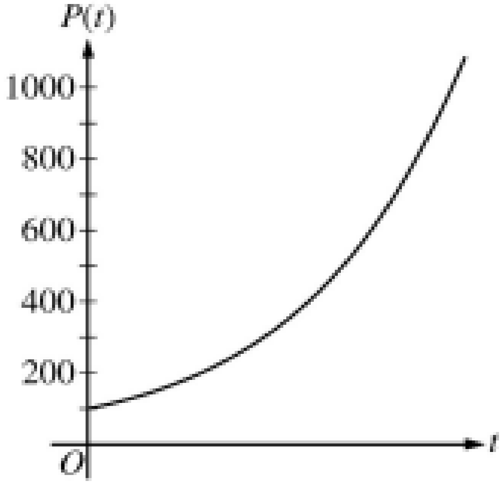
(B)



(C)



(D)



6. Which of the following series is absolutely convergent?

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n}$

(B) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

(C) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$

(D) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{2}\right)^{n}$

7. Which of the following series cannot be shown to converge using the limit comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$?

(A) $\sum_{n=1}^{\infty} \frac{4}{3n^2 - n}$

(B) $\sum_{n=1}^{\infty} \frac{15}{\sqrt{n^4} + 5}$

(C) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(D) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

8. The third-degree Taylor polynomial for the function f about $x = 0$ is $T(x) = 3 - 4x + 2x^2 - 3x^3$. Which of the following tables gives the values of f and its first three derivatives at $x = 0$?

(a)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	-8	6	-12

(b)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	-4	2	-3

(c)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	-4	4	-18

(d)

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	3	-4	4	-9

9. What is the interval of convergence for the power series $\sum_{n=1}^{\infty}(-1)^n \frac{n+2}{n \cdot 3^n}(x-4)^n$?

(A) $-3 < x < 3$

(B) $-3 < x \leq 3$

(C) $1 < x < 7$

(D) $1 < x \leq 7$

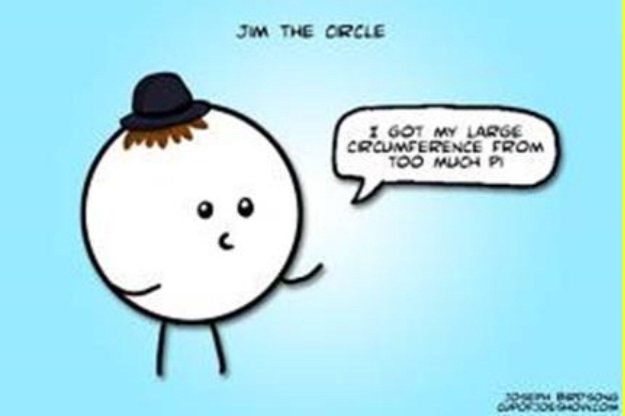
10. For time $t \geq 0$ seconds, the position of an object traveling along a curve in the *xy*-plane is given by the parametric equations $x(t)$ and $y(t)$, where $\frac{dx}{dt} = t^2 + 3$ and $\frac{dy}{dt} = t^3 + t$. At what time *t* is the speed of the object 10 units per second?

(A) 1.675

(B) 1.813

(C) 4.217

(D) 10.191



11. A particle moving in the xy-plane has velocity vector given by $v(t) = \left\langle e^{\sin t}, 5t^2 \right\rangle$ for time $t \geq 0$. What is the magnitude of the displacement of the particle between time $t = 1$ and $t = 2$?

(A) 3.778

(B) 11.954

(C) 11.992

(D) 15.001

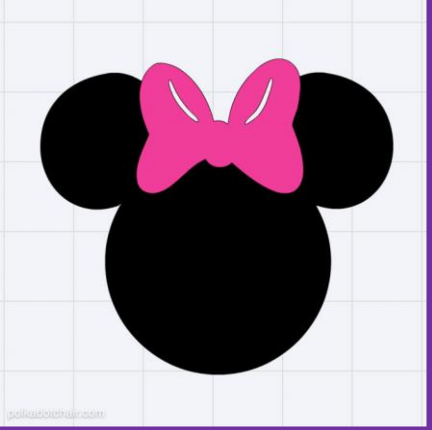
12. Consider the series $\sum_{n=0}^{\infty}(-1)^n a_n$, where $a_n > 0$ for all n. Which of the following conditions guarantees that the series converges?

(A) $\lim_{n\to\infty} a_n = 0$

(B) $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$

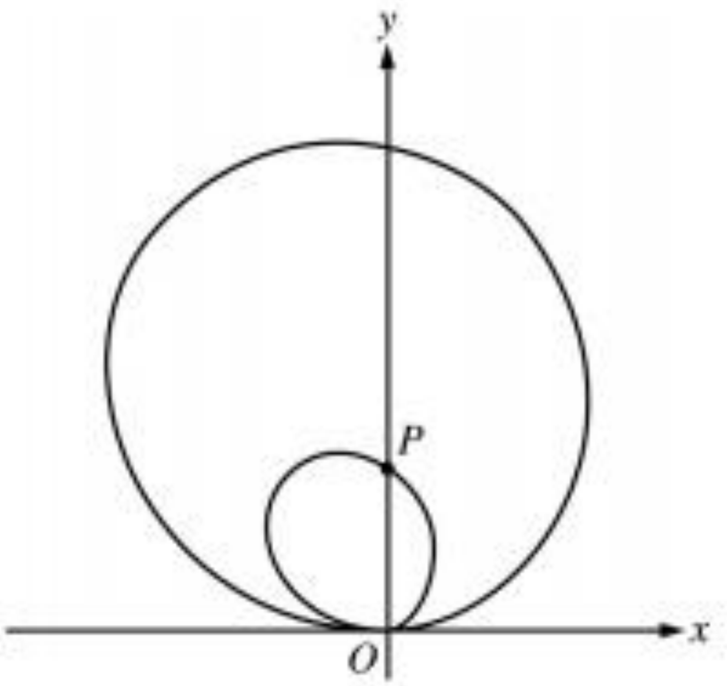
(C) $a_{n+1} < a_n$ for all n

(D) $\int_0^{\infty} f(x)\,dx$ converges, where $f(n) = a_n$ for all n



FREE RESPONCE

A graphing calculator is required for problems on this part of the exam.



1. Let r be the function given by $r(\theta) = 3\theta \sin \theta$ for $0 \le \theta \le 2\pi$. The graph of r in polar coordinates consists of two loops, as shown in the figure above. Point P is on the graph of r and the y-axis.

   (A) Find the rate of change of the x-coordinate with respect to θ at the point P.

   (B) Find the area of the region between the inner and outer loops of the graph.

   (C) The function r satisfies $\frac{dr}{d\theta} = 3\sin\theta + 3\theta\cos\theta$. For $0 \le \theta \le 2\pi$, find the value of θ that gives the point on the graph that is farthest from the origin. Justify your answer.

2. Consider the function f given by $f(x) = xe^{-2x}$ for all $x \geq 0$.

(A) Find $\lim\limits_{x \to \infty} f(x)$.

(B) Find the maximum value of f for $x \geq 0$. Justify your answer.

(C) Evaluate $\int_0^{\infty} f(x)\,dx$, or show that the integral diverges.

3. The function f is defined by the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^n}{3^n(n+1)} = 1 + \frac{x-2}{3 \cdot 2} + \frac{(x-2)^2}{3^2 \cdot 3} + \frac{(x-2)^3}{3^3 \cdot 4} + \cdots + \frac{(x-2)^n}{3^n(n+1)} + \cdots$$

for all real numbers x for which the series converges.

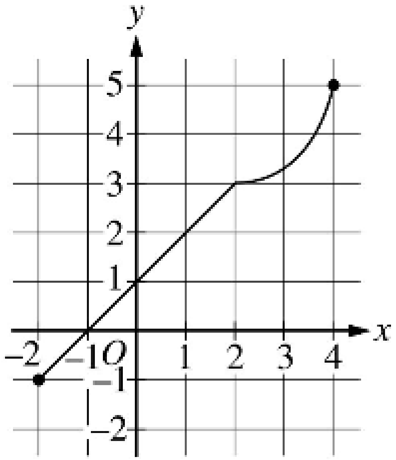
(A) Determine the interval of convergence of the power series for f. Show the work that leads to your answer.

(B) Find the value of $f'''(2)$.

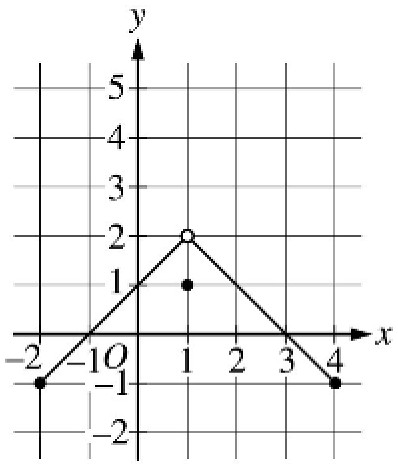
(C) Use the first three nonzero terms of the power series for f to approximate $f(1)$. Use the alternating series error bound to show that this approximation differs from $f(1)$ by less than $\frac{1}{100}$.

Multiple Choice: Section I, Part A

A calculator may not be used on questions on this part of the exam.



Graph of f



Graph of g

1. The graphs of the functions f and g are shown above. The value of $\lim_{x \to 1} f(g(x))$ is

(A) 1

(B) 2

(C) 3

(D) nonexistent

2. $\lim_{x \to 0} \dfrac{7x - \sin x}{x^2 + \sin(3x)} =$

(A) 6

(B) 2

(C) 1

(D) 0

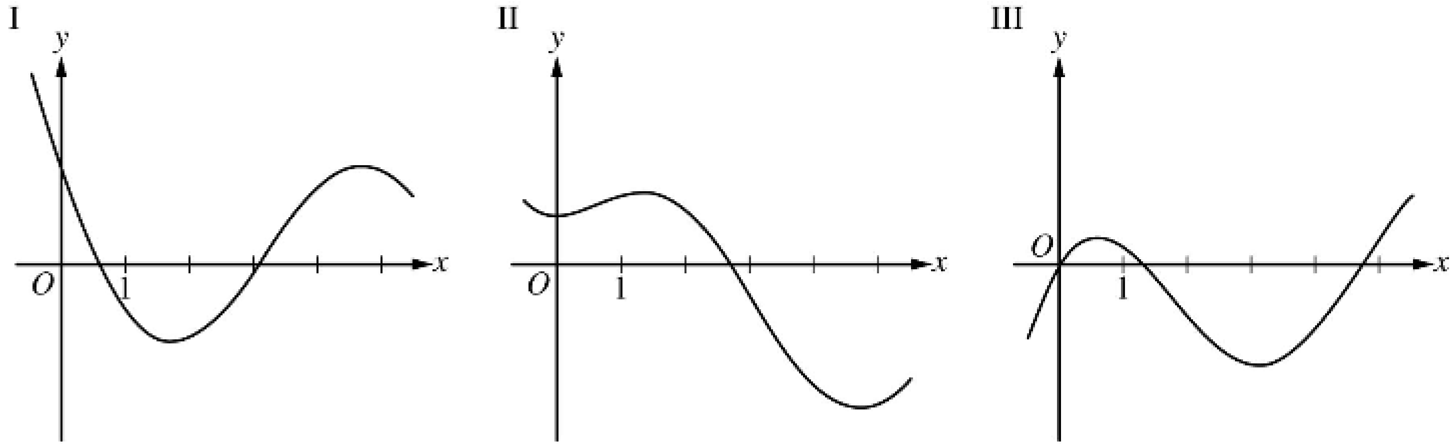
3. If $f(x) = \sin(\ln(2x))$, then $f'(x) =$

(A) $\dfrac{\sin(\ln(2x))}{2x}$

(B) $\dfrac{\cos(\ln(2x))}{x}$

(C) $\dfrac{\cos(\ln(2x))}{2x}$

(D) $\cos\left(\dfrac{1}{2x}\right)$



4. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	II	I	III
(C)	II	III	I
(D)	III	I	II

5. The local linear approximation to the function g at $x = \frac{1}{2}$ is $y = 4x + 1$. What is the value of $g\left(\frac{1}{2}\right) + g'\left(\frac{1}{2}\right)$?

(A) 4

(B) 5

(C) 6

(D) 7

6. For time $t \geq 0$, the velocity of a particle moving along the x-axis is given by $v(t) = (t-5)(t-2)^2$. At what values of t is the acceleration of the particle equal to 0?

(A) 2 only

(B) 4 only

(C) 2 and 4

(D) 2 and 5

7. The cost, in dollars, to shred the confidential documents of a company is modeled by C, a differentiable function of the weight of documents in pounds. Of the following, which is the best interpretation of $C'(500) = 80$?

(A) The cost to shred 500 pounds of documents is \$80.

(B) The average cost to shred documents is $\frac{80}{500}$ dollar per pound.

(C) Increasing the weight of documents by 500 pounds will increase the cost to shred the documents by approximately \$80.

(D) The cost to shred documents is increasing at a rate of \$80 per pound when the weight of the documents is 500 pounds.

8. Which of the following integral expressions is equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\sqrt{1 + \frac{3k}{n}} \cdot \frac{1}{n} \right)$?

(A) $\int_0^1 \sqrt{1 + 3x}\, dx$

(B) $\int_0^3 \sqrt{1 + x}\, dx$

(C) $\int_1^4 \sqrt{x}\, dx$

(D) $\frac{1}{3} \int_0^3 \sqrt{x}\, dx$

9. $f(x) = \begin{cases} x & \text{for } x < 2 \\ 3 & \text{for } x \geq 2 \end{cases}$

If f is the function defined above, then $\int_{-1}^{4} f(x)\,dx$ is

(A) $\frac{9}{2}$

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) undefined

10. $\int e^{x} \cos\left(e^{x}+1\right) dx =$

(A) $\sin\left(e^{x}+1\right)+C$

(B) $e^{x} \sin\left(e^{x}+1\right)+C$

(C) $e^{x} \sin\left(e^{x}+x\right)+C$

(D) $\frac{1}{2} \cos^{2}\left(e^{x}+1\right)+C$

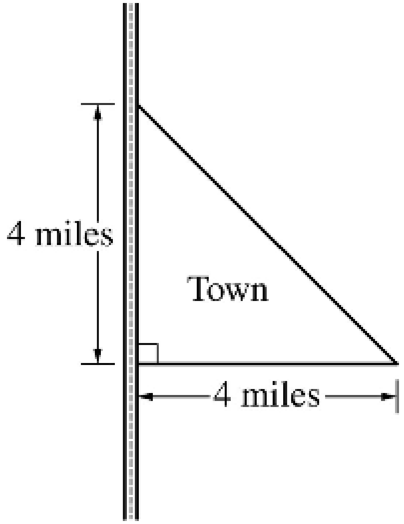
11. At time t, a population of bacteria grows at the rate of $5e^{0.2t} + 4t$ grams per day, where t is measured in days. By how many grams has the population grown from time $t = 0$ days to $t = 10$ days?

(A) $5e^2 + 40$

(B) $5e^2 + 195$

(C) $25e^2 + 175$

(D) $25e^2 + 375$



12. The right triangle shown in the figure above represents the boundary of a town that is bordered by a highway. The population density of the town at a distance of x miles from the highway is modeled by $D(x) = \sqrt{x+1}$, where $D(x)$ is measured in thousands of people per square mile. According to the model, which of the following expressions gives the total population, in thousands, of the town?

(A) $\int_0^4 \sqrt{x+1}\, dx$

(B) $\int_0^4 8\sqrt{x+1}\, dx$

(C) $\int_0^4 x\sqrt{x+1}\, dx$

(D) $\int_0^4 (4-x)\sqrt{x+1}\, dx$

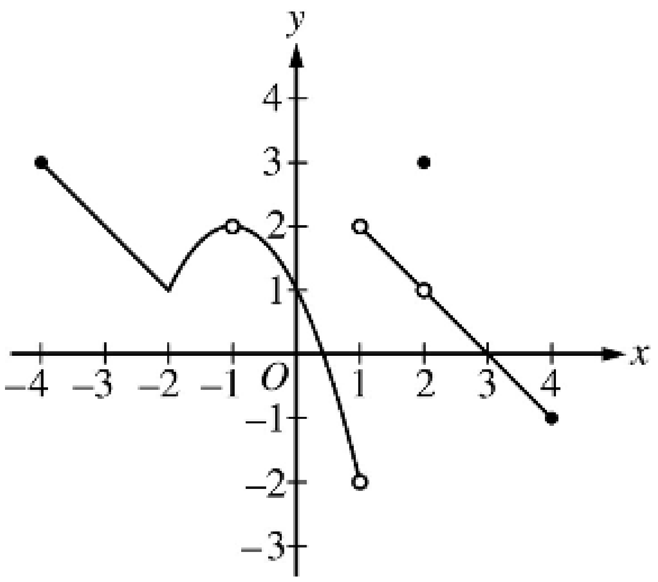
13. Which of the following is the solution to the differential equation $\frac{dy}{dx} = y\sec^2 x$ with the initial condition $y\left(\frac{\pi}{4}\right) = -1$?

(A) $y = -e^{\tan x}$

(B) $y = -e^{(-1+\tan x)}$

(C) $y = -e^{\left(\sec^3 x - 2\sqrt{2}\right)/3}$

(D) $y = -\sqrt{2\tan x - 1}$



Graph of f

14. The graph of the function f is shown in the figure above. For how many values of x in the open interval $(-4, 4)$ is f discontinuous?

(A) one

(B) two

(C) three

(D) four

15.

x	0	1	2
$f(x)$	5	2	-7
$f'(x)$	-2	-5	-14

The table above gives selected values of a differentiable and decreasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(2)$?

(A) $-\frac{1}{5}$

(B) $-\frac{1}{14}$

(C) $\frac{1}{5}$

(D) 5

16. The derivative of the function f is given by $f'(x) = -\frac{x}{3} + \cos\left(x^2\right)$. At what values of x does f have a relative minimum on the interval $0 < x < 3$?

(A) 1.094 and 2.608

(B) 1.798

(C) 2.372

(D) 2.493

17. The second derivative of a function g is given by $g''(x) = 2^{-x^2} + \cos x + x$. For $-5 < x < 5$, on what open intervals is the graph of g concave up?

(A) $-5 < x < -1.016$ only

(B) $-1.016 < x < 5$ only

(C) $0.463 < x < 2.100$ only

(D) $-5 < x < 0.463$ and $2.100 < x < 5$

18. The temperature, in degrees Fahrenheit $(°F)$, of water in a pond is modeled by the function H given by $H(t) = 55 - 9\cos\left(\frac{2\pi}{365}(t+10)\right)$, where t is the number of days since January 1 $(t = 0)$. What is the instantaneous rate of change of the temperature of the water at time $t = 90$ days?

(A) 0.114°F/day

(B) 0.153°F/day

(C) 50.252°F/day

(D) 56.350°F/day

19.

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivative at selected values of x. If h is the function given by $h(x) = f(2x)$, which of the following statements must be true?

(I) h is increasing on $2 < x < 4$.

(II) There exists c, where $0 < c < 4$, such that $h(c) = 12$.

(III) There exists c, where $0 < c < 2$, such that $h'(c) = 3$.

(A) II only

(B) I and III only

(C) II and III only

(D) I, II, and III

20. Let h be the function defined by $h(x) = \dfrac{1}{\sqrt{x^5+1}}$. If g is an antiderivative of h and $g(2) = 3$, what is the value of $g(4)$?

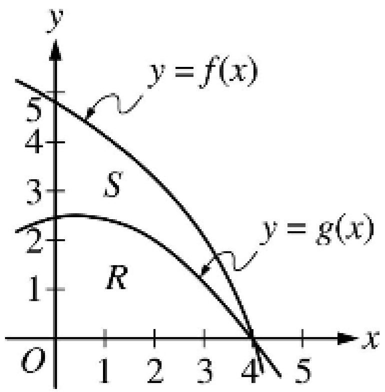
(A) -0.020

(B) 0.152

(C) 3.031

(D) 3.152

A graphing calculator is required for problems on this part of the exam.



1. Let R be the region in the first quadrant bounded by the graph of g, and let S be the region in the first quadrant between the graphs of f and g, as shown in the figure above. The region in the first quadrant bounded by the graph of f and the coordinate axes has area 12.142. The function g is given by $g(x) = (\sqrt{x+6})\cos\left(\frac{\pi x}{8}\right)$, and the function f is not explicitly given. The graphs of f and g intersect at the point $(4, 0)$.

   (A) Find the area of S.

   (B) A solid is generated when S is revolved about the horizontal line $y = 5$. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

   (C) Region R is the base of an art sculpture. At all points in R at a distance x from the y-axis, the height of the sculpture is given by $h(x) = 4 - x$. Find the volume of the art sculpture.

2.

t (minutes)	0	3	5	6	9
$r(t)$ (rotations per minute)	72	95	112	77	50

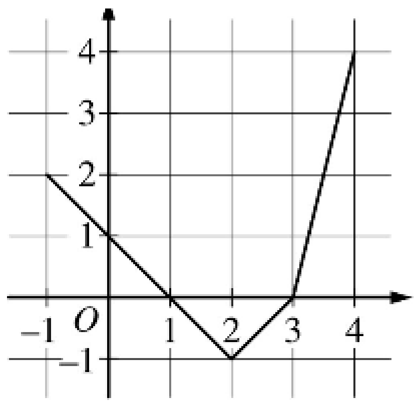
Rochelle rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Rochelle's ride is modeled by a differentiable function r for $0 \le t \le 9$ minutes. Values of $r(t)$ for selected values of t are shown in the table above.

(A) Estimate $r'(4)$. Show the computations that lead to your answer. Indicate units of measure.

(B) Is there a time t, for $3 \le t \le 5$, at which $r(t)$ is 106 rotations per minute? Justify your answer.

(C) Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^9 r(t)\,dt$. Using correct units, explain the meaning of $\int_0^9 r(t)\,dt$ in the context of the problem.

(D) Sarah also rode a stationary bicycle. The number of rotations per minute of the wheel of the stationary bicycle at time t minutes during Sarah's ride is modeled by the function s, defined by $s(t) = 40 + 20\pi \sin\left(\frac{\pi t}{18}\right)$ for $0 \le t \le 9$ minutes. Find the average number of rotations per minute of the wheel of the stationary bicycle for $0 \le t \le 9$ minutes.



Graph of f

3. Let f be a continuous function defined on the closed interval $-1 \le x \le 4$. The graph of f, consisting of three line segments, is shown above. Let g be the function defined by $g(x) = 5 + \int_2^x f(t)\,dt$ for $-1 \le x \le 4$.

(A) Find $g(4)$.

(B) On what intervals is g increasing? Justify your answer.

(C) On the closed interval $-1 \le x \le 4$, find the absolute minimum value of g and find the absolute maximum value of g. Justify your answers.

(D) Let $h(x) = x \cdot g(x)$. Find $h'(2)$.