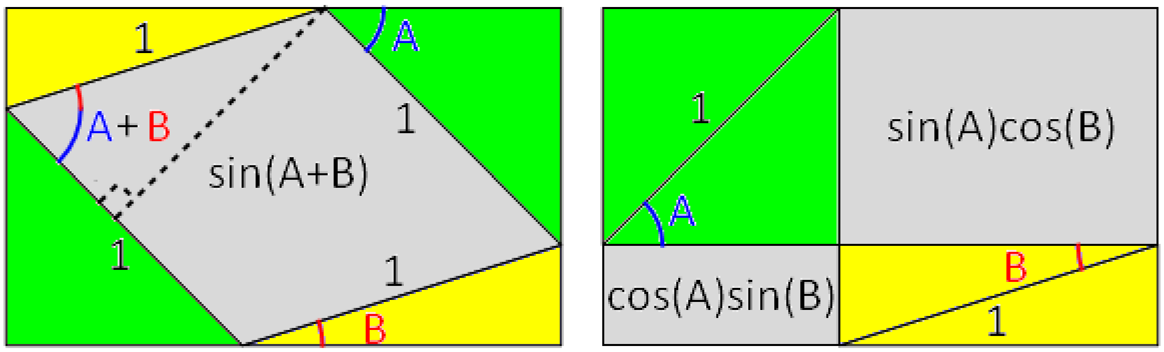
Part 2: Trig Identities Made Easy

Trig identities are often among the hardest mathematical formulas to reliably commit to memory. Their proofs often involve subtle trickery or difficult to recreate arguments. One “proof without words” for the angle addition formula for Sines is shown here.



Fortunately, Taylor Series give us another way to approach these formulas!

1. Fill in the first seven coefficients for the Maclaurin Series for $e^{\blacksquare}$.

$e^{\blacksquare}$ = _______ + _______ $\blacksquare$ + _______ $\blacksquare^2$ + _______ $\blacksquare^3$ + _______ $\blacksquare^4$ + _______ $\blacksquare^5$ + _______ $\blacksquare^6$ + ...

2. Now replace ‘■‘ with ‘iθ’, where i is the imaginary unit ($i^2 = -1$) to get a series for $e^{i\theta}$, simplifying all of the terms with a power of i greater than 1.

$e^{i\theta}$ = _______ + _______ θ + _______ θ^2 + _______ θ^3 + _______ θ^4 + _______ θ^5 + _______ θ^6 + ...

3. Write the Maclaurin Series for cos(θ) and sin(θ).

$\cos(\theta)$ = _______ + _______ θ + _______ θ^2 + _______ θ^3 + _______ θ^4 + _______ θ^5 + _______ θ^6 + ...

$\sin(\theta)$ = _______ + _______ θ + _______ θ^2 + _______ θ^3 + _______ θ^4 + _______ θ^5 + _______ θ^6 + ...

4. Carefully examining the power series in (2) and (3), identify a relationship between $e^{i\theta}$, cos(θ) and sin(θ).

$e^{i\theta}$ = ____________ + i ______________

This identity is known as *Euler’s Identity* and it provides a deep link between the seemingly very different exponential function and the elementary trigonometric functions. One side effect is that the previously complicated *trig identities* now follow from the much simpler identities for *exponentials*.

5. Fill in the blank to complete the rule for multiplying exponentials: $z^m * z^n = z^{\_\_\_\_\_\_}$.

6. Write $e^{i(A+B)}$ in terms of sines and cosines by using Euler's Identity.

7. Write $e^{iA} * e^{iB}$ in terms of sines and cosines by applying Euler's Identity to e^{iA} and e^{iB} separately and then FOILing the results together. Please simplify any powers of i.

8. According to (5), the quantities you wrote down in (6) and (7) must be equal. Now, two complex-valued quantities are equal *if and only if* their real parts (the terms not involving i) are equal and, separately, their imaginary parts are equal.

a. Identify the real parts of the quantities in (6) and (7) and set them equal here.

____________________________________ = ___________________________________.

b. Identify the imaginary parts of the quantities in (6) and (7) and set them equal here. You do not have to include the parameter i on either side of this equation – just the terms that we *multiply by i*.

____________________________________ = ___________________________________.

The identities in (8a) and (8b) should be recognizable as the angle addition formulas for cosine and sine!! We have now derived these formulas without ever drawing a single triangle!!