Part 1: Approximating Definite Integrals

Earlier in the course, we developed several methods for numerically approximating the area under a curve: the midpoint rule and the trapezoid rule. Taylor Series provide an alternate route to such approximations. In this activity, we hope to approximate the value of $\int_0^1 \sin(x^2)dx$.

Sadly, the integrand here does not have an anti-derivative that can be expressed in terms of elementary functions. However, if we can express the integrand as a power series, then we will also be able to write a power series for its anti-derivative.

1. Find the Maclaurin Series for sin(■).

2. Use your result in (1), replacing '■' with 'x^2', to write down a power series for $\sin(x^2)$.

3. Anti-differentiate the power series in (2) term-by-term to find a power series for $\int \sin(x^2)dx$.

4. Use the Fundamental Theorem of Calculus, along with the power series in (3), to write down a series that converges to the number $\int_0^1 \sin(x^2)dx$.

5. Use the first four terms of the series in (4) to approximate $\int_0^1 \sin(x^2)dx$. Is this an underestimate or an overestimate? According to the Alternating Series Theorem, what is the maximum possible error of this approximation?