

Harold's Calculus Notes

Cheat Sheet

15 December 2015

AP Calculus

Limits	
Definition of Limit Let f be a function defined on an open interval containing c and let L be a real number. The statement: $\lim_{x \rightarrow a} f(x) = L$ means that for each $\epsilon > 0$ there exists a $\delta > 0$ such that if $0 < x - a < \delta$, then $ f(x) - L < \epsilon$ <p>Tip : Direct substitution: Plug in $f(a)$ and see if it provides a legal answer. If so then $L = f(a)$.</p>	
The Existence of a Limit The limit of $f(x)$ as x approaches a is L if and only if: <p>Definition of Continuity</p> A function f is continuous at c if for every $\epsilon > 0$ there exists a $\delta > 0$ such that $ x - c < \delta$ and $ f(x) - f(c) < \epsilon$. <p>Tip: Rearrange $f(x) - f(c)$ to have $(x - c)$ as a factor. Since $x - c < \delta$ we can find an equation that relates both δ and ϵ together.</p>	$\lim_{x \rightarrow a^-} f(x) = L$ $\lim_{x \rightarrow a^+} f(x) = L$ <p>Prove that $f(x) = x^2 - 1$ is a continuous function.</p> $ \begin{aligned} & f(x) - f(c) \\ &= (x^2 - 1) - (c^2 - 1) \\ &= x^2 - 1 - c^2 + 1 \\ &= x^2 - c^2 \\ &= (x + c)(x - c) \\ &= (x + c) (x - c) \end{aligned} $ <p>Since $(x + c) \leq 2c$</p> $ f(x) - f(c) \leq 2c (x - c) < \epsilon$ <p>So given $\epsilon > 0$, we can choose $\delta = \left \frac{1}{2c} \right \epsilon > 0$ in the Definition of Continuity. So substituting the chosen δ for $(x - c)$ we get:</p> $ f(x) - f(c) \leq 2c \left(\left \frac{1}{2c} \right \epsilon \right) = \epsilon$ <p>Since both conditions are met, $f(x)$ is continuous.</p>
Two Special Trig Limits	$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

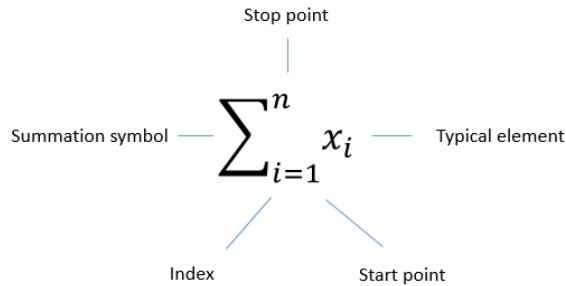
Derivatives	(See Larson's 1-pager of common derivatives)
Definition of a Derivative of a Function Slope Function	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$
Notation for Derivatives	$f'(x), f^{(n)}(x), \frac{dy}{dx}, y', \frac{d}{dx}[f(x)], D_x[y]$
The Constant Rule	$\frac{d}{dx}[c] = 0$
The Power Rule	$\frac{d}{dx}[x^n] = nx^{n-1}$ $\frac{d}{dx}[x] = 1 \text{ (think } x = x^1 \text{ and } x^0 = 1\text{)}$
The General Power Rule	$\frac{d}{dx}[u^n] = nu^{n-1} u' \text{ where } u = u(x)$
The Constant Multiple Rule	$\frac{d}{dx}[cf(x)] = cf'(x)$
The Sum and Difference Rule	$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
Position Function	$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$
Velocity Function	$v(t) = s'(t) = gt + v_0$
Acceleration Function	$a(t) = v'(t) = s''(t)$
Jerk Function	$j(t) = a'(t) = v'''(t) = s^{(3)}(t)$
The Product Rule	$\frac{d}{dx}[fg] = fg' + g f'$
The Quotient Rule	$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$
The Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$ $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
Exponentials (e^x, a^x)	$\frac{d}{dx}[e^x] = e^x, \quad \frac{d}{dx}[a^x] = (\ln a) a^x$
Logarithms ($\ln x, \log_a x$)	$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad \frac{d}{dx}[\log_a x] = \frac{1}{(\ln a) x}$
Sine	$\frac{d}{dx}[\sin(x)] = \cos(x)$
Cosine	$\frac{d}{dx}[\cos(x)] = -\sin(x)$
Tangent	$\frac{d}{dx}[\tan(x)] = \sec^2(x)$
Secent	$\frac{d}{dx}[\sec(x)] = \sec(x) \tan(x)$
Cosecent	$\frac{d}{dx}[\csc(x)] = -\csc(x) \cot(x)$
Cotangent	$\frac{d}{dx}[\cot(x)] = -\csc^2(x)$

Applications of Differentiation	
Rolle's Theorem f is continuous on the closed interval [a,b], and f is differentiable on the open interval (a,b).	If $f(a) = f(b)$, then there exists at least one number c in (a,b) such that $f'(c) = 0$.
Mean Value Theorem If f meets the conditions of Rolle's Theorem, then	$f'(c) = \frac{f(b) - f(a)}{b - a}$ $f(b) = f(a) + (b - a)f'(c)$ Find ' c '.
L'Hôpital's Rule	$\text{If } \lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} \frac{P(x)}{Q(x)} =$ $\left\{ \frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 1^\infty, 0^0, \infty^0, \infty - \infty \right\}, \text{ but not } \{0^\infty\},$ $\text{then } \lim_{x \rightarrow c} \frac{P(x)}{Q(x)} = \lim_{x \rightarrow c} \frac{P'(x)}{Q'(x)} = \lim_{x \rightarrow c} \frac{P''(x)}{Q''(x)} = \dots$
Graphing with Derivatives	
Test for Increasing and Decreasing Functions	<ol style="list-style-type: none"> If $f'(x) > 0$, then f is increasing (slope up) ↗ If $f'(x) < 0$, then f is decreasing (slope down) ↘ If $f'(x) = 0$, then f is constant (zero slope) →
The First Derivative Test	<ol style="list-style-type: none"> If $f'(x)$ changes from - to + at c, then f has a <i>relative minimum</i> at $(c, f(c))$ If $f'(x)$ changes from + to - at c, then f has a <i>relative maximum</i> at $(c, f(c))$ If $f'(x)$ is + c + or - c -, then $f(c)$ is neither
The Second Derivative Test Let $f'(c)=0$, and $f''(x)$ exists, then	<ol style="list-style-type: none"> If $f''(x) > 0$, then f has a relative minimum at $(c, f(c))$ If $f''(x) < 0$, then f has a relative maximum at $(c, f(c))$ If $f''(x) = 0$, then the test fails (See 1st derivative test)
Test for Concavity	<ol style="list-style-type: none"> If $f''(x) > 0$ for all x, then the graph is concave up U If $f''(x) < 0$ for all x, then the graph is concave down ∩
Points of Inflection Change in concavity	<p>If $(c, f(c))$ is a point of inflection of f, then either</p> <ol style="list-style-type: none"> $f''(c) = 0$ or f'' does not exist at $x = c$.
Analyzing the Graph of a Function	
x-Intercepts (Zeros or Roots)	$f(x) = 0$
y-Intercept	$f(0) = y$
Domain	Valid x values
Range	Valid y values
Continuity	No division by 0, no negative square roots or logs
Vertical Asymptotes (VA)	$x =$ division by 0 or undefined
Horizontal Asymptotes (HA)	$\lim_{x \rightarrow \infty^-} f(x) \rightarrow y$ and $\lim_{x \rightarrow \infty^+} f(x) \rightarrow y$
Infinite Limits at Infinity	$\lim_{x \rightarrow \infty^-} f(x) \rightarrow \infty$ and $\lim_{x \rightarrow \infty^+} f(x) \rightarrow \infty$
Differentiability	Limit from both directions arrives at the same slope
Relative Extrema	Create a table with <i>domains</i> , $f(x)$, $f'(x)$, and $f''(x)$
Concavity	If $f''(x) \rightarrow +$, then cup up U If $f''(x) \rightarrow -$, then cup down ∩
Points of Inflection	$f''(x) = 0$ (concavity changes)

Approximating with Differentials	
Newton's Method Finds zeros of f , or finds c if $f(c) = 0$.	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
Tangent Line Approximations	$y = mx + b$ $y = f'(c)(x - c) + f(c)$
Function Approximations with Differentials	$f(x + \Delta x) \approx f(x) + dy = f(x) + f'(x) dx$
Related Rates	<p>Steps to solve:</p> <ol style="list-style-type: none"> Identify the known variables and rates of change. $(x = 2 \text{ m}; y = -3 \text{ m}; x' = 4 \frac{\text{m}}{\text{s}}; y' = ?)$ Construct an equation relating these quantities. $(x^2 + y^2 = r^2)$ Differentiate both sides of the equation. $(2xx' + 2yy' = 0)$ Solve for the desired rate of change. $(y' = -\frac{x}{y} x')$ Substitute the known rates of change and quantities into the equation. $(y' = -\frac{2}{-3} \cdot 4 = \frac{8}{3} \frac{\text{m}}{\text{s}})$

Integration	
Basic Integration Rules Integration is the “inverse” of differentiation, and vice versa.	$\int f'(x) dx = f(x) + C$ $\frac{d}{dx} \int f(x) dx = f(x)$
$f(x) = 0$	$\int 0 dx = C$
$f(x) = k = kx^0$	$\int k dx = kx + C$
The Constant Multiple Rule	$\int k f(x) dx = k \int f(x) dx$
The Sum and Difference Rule	$\int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$
The Power Rule $f(x) = kx^n$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \text{ where } n \neq -1$ $If n = -1, \text{ then } \int x^{-1} dx = \ln x + C$
The General Power Rule	If $u = g(x)$, and $u' = \frac{d}{dx} g(x)$ then $\int u^n u' dx = \frac{u^{n+1}}{n+1} + C, \text{ where } n \neq -1$
Reimann Sum	$\sum_{i=1}^n f(c_i) \Delta x_i, \quad \text{where } x_{i-1} \leq c_i \leq x_i$ $\ \Delta\ = \Delta x = \frac{b-a}{n}$
Definition of a Definite Integral Area under curve	$\lim_{\ \Delta\ \rightarrow 0} \sum_{i=1}^n f(c_i) \Delta x_i = \int_a^b f(x) dx$
Swap Bounds	$\int_a^b f(x) dx = - \int_b^a f(x) dx$
Additive Interval Property	$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$
The Fundamental Theorem of Calculus	$\int_a^b f(x) dx = F(b) - F(a)$
The Second Fundamental Theorem of Calculus (See Harold's Fundamental Theorem of Calculus Cheat Sheet)	$\frac{d}{dx} \int_a^x f(t) dt = f(x)$ $\frac{d}{dx} \int_a^{g(x)} f(t) dt = f(g(x))g'(x)$ $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt = f(h(x))h'(x) - f(g(x))g'(x)$
Mean Value Theorem for Integrals	$\int_a^b f(x) dx = f(c)(b-a) \quad \text{Find 'c'}$
The Average Value for a Function	$\frac{1}{b-a} \int_a^b f(x) dx$

Summation Formulas



$$\sum_{i=1}^4 x_i = x_1 + x_2 + x_3 + x_4$$

Sum of Powers

$$\begin{aligned}\sum_{i=1}^n c &= cn \\ \sum_{i=1}^n i &= \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \\ \sum_{i=1}^n i^3 &= \left(\sum_{i=1}^n i\right)^2 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4} \\ \sum_{i=1}^n i^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30} \\ \sum_{i=1}^n i^5 &= \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12} \\ \sum_{i=1}^n i^6 &= \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42} \\ \sum_{i=1}^n i^7 &= \frac{n^2(n+1)^2(3n^4+6n^3-n^2-4n+2)}{24} \\ S_k(n) &= \sum_{i=1}^n i^k = \frac{(n+1)^{k+1}}{k+1} - \frac{1}{k+1} \sum_{r=0}^{k-1} \binom{k+1}{r} S_r(n)\end{aligned}$$

Misc. Summation Formulas

$$\begin{aligned}\sum_{i=1}^n i(i+1) &= \sum_{i=1}^n i^2 + \sum_{i=1}^n i = \frac{n(n+1)(n+2)}{3} \\ \sum_{i=1}^n \frac{1}{i(i+1)} &= \frac{n}{n+1} \\ \sum_{i=1}^n \frac{1}{i(i+1)(i+2)} &= \frac{n(n+3)}{4(n+1)(n+2)}\end{aligned}$$

Integration Methods	
1. Memorized	See Larson's 1-pager of common integrals
2. U-Substitution	$\int f(g(x))g'(x)dx = F(g(x)) + C$ Set $u = g(x)$, then $du = g'(x) dx$ $\int f(u) du = F(u) + C$ $u = \underline{\hspace{2cm}} \quad du = \underline{\hspace{2cm}} dx$
3. Integration by Parts	$\int u dv = uv - \int v du$ $u = \underline{\hspace{2cm}} \quad v = \underline{\hspace{2cm}}$ $du = \underline{\hspace{2cm}} \quad dv = \underline{\hspace{2cm}}$ <p>Pick 'u' using the LIATED Rule:</p> <p>L - Logarithmic: $\ln x, \log_b x, \text{etc.}$</p> <p>I - Inverse Trig.: $\tan^{-1} x, \sec^{-1} x, \text{etc.}$</p> <p>A - Algebraic: $x^2, 3x^{60}, \text{etc.}$</p> <p>T - Trigonometric: $\sin x, \tan x, \text{etc.}$</p> <p>E - Exponential: $e^x, 19^x, \text{etc.}$</p> <p>D - Derivative of: $\frac{dy}{dx}$</p>
4. Partial Fractions	$\int \frac{P(x)}{Q(x)} dx$ where $P(x)$ and $Q(x)$ are polynomials <p>Case 1: If degree of $P(x) \geq Q(x)$ then do long division first</p> <p>Case 2: If degree of $P(x) < Q(x)$ then do partial fraction expansion</p>
5a. Trig Substitution for $\sqrt{a^2 - x^2}$	$\int \sqrt{a^2 - x^2} dx$ Substitution: $x = a \sin \theta$ Identity: $1 - \sin^2 \theta = \cos^2 \theta$
5b. Trig Substitution for $\sqrt{x^2 - a^2}$	$\int \sqrt{x^2 - a^2} dx$ Substitution: $x = a \sec \theta$ Identity: $\sec^2 \theta - 1 = \tan^2 \theta$
5c. Trig Substitution for $\sqrt{x^2 + a^2}$	$\int \sqrt{x^2 + a^2} dx$ Substitution: $x = a \tan \theta$ Identity: $\tan^2 \theta + 1 = \sec^2 \theta$
6. Table of Integrals	CRC Standard Mathematical Tables book
7. Computer Algebra Systems (CAS)	TI-Nspire CX CAS Graphing Calculator TI-Nspire CAS iPad app
8. Numerical Methods	Riemann Sum, Midpoint Rule, Trapezoidal Rule, Simpson's Rule, TI-84
9. WolframAlpha	Google of mathematics. Shows steps. Free. www.wolframalpha.com

Numerical Methods	
Riemann Sum	$P_0(x) = \int_a^b f(x) dx = \lim_{\ P\ \rightarrow 0} \sum_{i=1}^n f(x_i^*) \Delta x_i$ <p>where $a = x_0 < x_1 < x_2 < \dots < x_n = b$ and $\Delta x_i = x_i - x_{i-1}$ and $\ P\ = \max\{\Delta x_i\}$</p> <p>Types:</p> <ul style="list-style-type: none"> • Left Sum (LHS) • Middle Sum (MHS) • Right Sum (RHS)
Midpoint Rule	$P_0(x) = \int_a^b f(x) dx \approx \sum_{i=1}^n f(\bar{x}_i) \Delta x =$ $\Delta x [f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + \dots + f(\bar{x}_n)]$ <p>where $\Delta x = \frac{b-a}{n}$</p> <p>and $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i) = \text{midpoint of } [x_{i-1}, x_i]$</p> <p>Error Bounds: $E_M \leq \frac{K(b-a)^3}{24n^2}$</p>
Trapezoidal Rule	$P_1(x) = \int_a^b f(x) dx \approx$ $\frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_3) + \dots + 2f(x_{n-1}) + f(x_n)]$ <p>where $\Delta x = \frac{b-a}{n}$</p> <p>and $x_i = a + i\Delta x$</p> <p>Error Bounds: $E_T \leq \frac{K(b-a)^3}{12n^2}$</p>
Simpson's Rule	$P_2(x) = \int_a^b f(x) dx \approx$ $\frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$ <p>Where n is even</p> <p>and $\Delta x = \frac{b-a}{n}$</p> <p>and $x_i = a + i\Delta x$</p> <p>Error Bounds: $E_S \leq \frac{K(b-a)^5}{180n^4}$</p>
TI-84 Plus	<p>[MATH] fnInt(f(x),x,a,b), [MATH] [1] [ENTER]</p> <p>Example: [MATH] fnInt(x^2,x,0,1) $\int_0^1 x^2 dx = \frac{1}{3}$</p>
TI-Nspire CAS	<p>[MENU] [4] Calculus [3] Integral [TAB] [TAB] [X] [^] [2] [TAB] [TAB] [X] [ENTER]</p>

Partial Fractions	(http://en.wikipedia.org/wiki/Partial_fraction_decomposition)
Condition	$f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials and degree of $P(x) < Q(x)$
Case I: Simple linear (1st degree)	$\frac{A}{(ax + b)}$
Case II: Multiple degree linear (1st degree)	$\frac{A}{(ax + b)} + \frac{B}{(ax + b)^2} + \frac{C}{(ax + b)^3}$
Case III: Simple quadratic (2nd degree)	$\frac{Ax + B}{(ax^2 + bx + c)}$
Case IV: Multiple degree quadratic (2nd degree)	$\frac{Ax + B}{(ax^2 + bx + c)} + \frac{Cx + D}{(ax^2 + bx + c)^2} + \frac{Ex + F}{(ax^2 + bx + c)^3}$
Typical Solution for Cases I & II	$\int \frac{a}{x + b} dx = a \ln x + b + C$
Typical Solution for Cases III & IV	$\int \frac{a}{x^2 + b^2} dx = a \tan^{-1}\left(\frac{x}{b}\right) + C$

Series	Arithmetic	Geometric
Sequence	$\lim_{n \rightarrow \infty} a_n = L$ (Limit) Example: $(a_n, a_{n+1}, a_{n+2}, \dots)$	
Summation Notation	$S_n = \sum_{k=1}^n a_k$	$S_n = \sum_{k=0}^{n-1} a_0 r^k = \sum_{k=1}^n a_0 r^{k-1}$
Summation Expanded	$S_n = a_1 + a_2 + \dots + a_{n-1} + a_n$ (Partial Sum)	$S_n = a_0 + a_0 r + a_0 r^2 + \dots + a_0 r^{n-1}$
Sum of n Terms (finite series)	$S_n = n \left(\frac{a_1 + a_n}{2} \right)$ $S_n = \frac{n}{2} (2a_1 + (n - 1)d)$	$S_n = a_0 \frac{(1 - r^n)}{1 - r}$
Sum of ∞ Terms (infinite series)	$S \rightarrow \infty$	$S = \lim_{n \rightarrow \infty} \frac{a(1 - r^n)}{1 - r} = \frac{a}{1 - r}$ only if $ r < 1$ where r is the radius of convergence and $(-r, r)$ is the interval of convergence
Recursive nth Term	$a_n = a_{n-1} + d$	$a_n = a_{n-1} r$
Explicit nth Term	$a_n = a_1 + d(n - 1)$	$a_n = a_0 r^{n-1}$

Convergence Tests	(See Harold's Series Convergence Tests Cheat Sheet)
Convergence Tests	<ol style="list-style-type: none"> 1. n^{th} Term 2. Geometric Series 3. p-Series 4. Alternating Series 5. Integral 6. Ratio 7. Root 8. Direct Comparison 9. Limit Comparison 10. Telescoping
Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$ <p>Converges if $\lim_{n \rightarrow \infty} b_n = L$</p> <p>Diverges if N/A</p> <p>Sum: $S = b_1 - L$</p>
Taylor Series	
Power Series	$\sum_{n=0}^{+\infty} a_n (x - c)^n = a_0 + a_1(x - c) + a_2(x - c)^2 + \dots$
Power Series About Zero	$\sum_{n=0}^{+\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots$
Maclaurin Series Taylor series about zero	$f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n$
Maclaurin Series with Remainder	$f(x) = P_n(x) + R_n(x)$ $= \sum_{n=0}^{+\infty} \frac{f^{(n)}(0)}{n!} x^n + \frac{f^{(n+1)}(x^*)}{(n+1)!} x^{n+1}$ <p>where $x \leq x^* \leq \max$ and $\lim_{x \rightarrow +\infty} R_n(x) = 0$</p>
Taylor Series	$f(x) \approx P_n(x) = \sum_{n=0}^{+\infty} \frac{f^{(n)}(c)}{n!} (x - c)^n$
Taylor Series with Remainder	$f(x) = P_n(x) + R_n(x)$ $= \sum_{n=0}^{+\infty} \frac{f^{(n)}(c)}{n!} (x - c)^n + \frac{f^{(n+1)}(x^*)}{(n+1)!} (x - c)^{n+1}$ <p>where $x \leq x^* \leq c$ and $\lim_{x \rightarrow +\infty} R_n(x) = 0$</p>

Common Series	
Exponential Functions	
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for all x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$
$a^x = e^{x \ln(a)} = \sum_{n=0}^{\infty} \frac{(x \ln(a))^n}{n!}$ for all x	$1 + x \ln(a) + \frac{(x \ln(a))^2}{2!} + \frac{(x \ln(a))^3}{3!} + \dots$
Natural Logarithms	
$\ln(1-x) = \sum_{n=1}^{\infty} \frac{x^n}{n}$ for $ x < 1$	$x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \dots$
$\ln(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{n}$ for $ x < 1$	$(x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \frac{(x-1)^4}{4} + \dots$
$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n$ for $ x < 1$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$
Geometric Series	
$\frac{1}{x} = \sum_{n=0}^{\infty} (-1)^n (x-1)^n$ for $0 < x < 2$	$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4 + \dots$
$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ for $ x < 1$	$1 - x + x^2 - x^3 + x^4 - \dots$
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ for $ x < 1$	$1 + x + x^2 + x^3 + x^4 + \dots$
$\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1}$ for $ x < 1$	$1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots$
$\frac{1}{(1-x)^3} = \sum_{n=2}^{\infty} \frac{(n-1)n}{2} x^{n-2}$ for $ x < 1$	$1 + 3x + 6x^2 + 10x^3 + 15x^4 + \dots$
Binomial Series	
$(1+x)^r = \sum_{n=0}^{+\infty} \binom{r}{n} x^n$ for $ x < 1$ and all complex r where $\binom{r}{n} = \prod_{k=1}^n \frac{r-k+1}{k}$ $= \frac{r(r-1)(r-2)\dots(r-n+1)}{n!}$	$1 + rx + \frac{r(r-1)}{2!} x^2 + \frac{r(r-1)(r-2)}{3!} x^3 + \dots$
Trigonometric Functions	
$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$ for all x	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$
$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$ for all x	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$

$\tan(x) = \sum_{n=1}^{\infty} \frac{B_{2n}(-4)^n(1-4^n)}{(2n)!} x^{2n-1}$ <p style="text-align: center;">for $x < \frac{\pi}{2}$</p>	$x + 2\frac{x^3}{3!} + 16\frac{x^5}{5!} + 272\frac{x^7}{7!} + 7936\frac{x^9}{9!} - \dots$ $= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{2}{945}x^9 - \dots$
<p>Bernoulli Numbers:</p> $B_0 = 1, B_1 = \frac{1}{2}, B_2 = \frac{1}{6}, B_4 = \frac{1}{30}, B_6 = \frac{1}{42},$ $B_8 = \frac{1}{30}, B_{10} = \frac{5}{66}, B_{12} = \frac{691}{2730}, B_{14} = \frac{7}{6}$	$\sec(x) = \sum_{n=0}^{\infty} \frac{(-1)^n E_{2n}}{(2n)!} x^{2n}$ <p style="text-align: center;">for $x < \frac{\pi}{2}$</p>
<p>Euler Numbers:</p> $E_0 = 1, E_2 = -1, E_4 = 5, E_6 = -61,$ $E_8 = 1,385, E_{10} = -50,521, E_{12} = 2,702,765$	$1 + \frac{x^2}{2!} + 5\frac{x^4}{4!} + 61\frac{x^6}{6!} + 1385\frac{x^8}{8!} + 50,521\frac{x^{10}}{10!} + \dots$
$\arcsin(x) = \sum_{n=0}^{\infty} \frac{(2n)!}{(2^n n!)^2 (2n+1)} x^{2n+1}$ <p style="text-align: center;">for $x \leq 1$</p>	$x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$
$\arccos(x) = \frac{\pi}{2} - \arcsin(x)$ <p style="text-align: center;">for $x \leq 1$</p>	$\frac{\pi}{2} - x - \frac{x^3}{2 \cdot 3} - \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} - \frac{1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} - \dots$
$\arctan(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} x^{2n+1}$ <p style="text-align: center;">for $x < 1, x \neq \pm i$</p>	$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$
Hyperbolic Functions	
$\sinh(x) = \frac{e^x - e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \text{ for all } x$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$
$\cosh(x) = \frac{e^x + e^{-x}}{2} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} \text{ for all } x$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$
$\tanh(x) = \sum_{n=1}^{\infty} \frac{B_{2n} 4^n (4^n - 1)}{(2n)!} x^{2n-1}$ <p style="text-align: center;">for $x < \frac{\pi}{2}$</p>	$x - 2\frac{x^3}{3!} + 16\frac{x^5}{5!} - 272\frac{x^7}{7!} + 7936\frac{x^9}{9!} - \dots$ $x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{2}{945}x^9 - \dots$
$\operatorname{arsinh}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(2^n n!)^2 (2n+1)} x^{2n+1}$ <p style="text-align: center;">for $x \leq 1$</p>	$x - \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} - \frac{1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$
$\operatorname{arccosh}(x) = \frac{\pi}{2} i - i \sum_{n=0}^{\infty} \frac{2^{-n}}{n! (2n+1)} x^{2n+1}$ <p style="text-align: center;">for $x \leq 1$</p>	$\frac{\pi i}{2} - i x - \frac{i x^3}{2 \cdot 3} - \frac{i \cdot 1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} - \frac{i \cdot 1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} - \dots$
$\operatorname{arctanh}(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)}$ <p style="text-align: center;">for $x < 1, x \neq \pm 1$</p>	$x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \dots$