



AP Calculus BC – AP Exam Review Project –Free Response Problems → Group 4

Year & #	2016 4	2015 5	2014 5	2011 1	2010 3	
National Average	4.68	6.21	4.45	5.24	3.89	
Type of Problem	2 <sup>nd</sup> Derivative, Differentials, & Limit	Partial Fractions, Max, & Min	Area, Volume, Perimeter	Parametric	Speed & Total Distance	WP WP

Consider the differential equation  $\frac{dy}{dx} = x^2 - \frac{1}{2}y$ .

- (a) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .
- (b) Let  $y = f(x)$  be the particular solution to the given differential equation whose graph passes through the point  $(-2, 8)$ . Does the graph of  $f$  have a relative minimum, a relative maximum, or neither at the point  $(-2, 8)$ ? Justify your answer.
- (c) Let  $y = g(x)$  be the particular solution to the given differential equation with  $g(-1) = 2$ . Find  $\lim_{x \rightarrow -1} \left( \frac{g(x) - 2}{3(x + 1)^2} \right)$ . Show the work that leads to your answer.
- (d) Let  $y = h(x)$  be the particular solution to the given differential equation with  $h(0) = 2$ . Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $h(1)$ .

Consider the function  $f(x) = \frac{1}{x^2 - kx}$ , where  $k$  is a nonzero constant. The derivative of  $f$  is given by

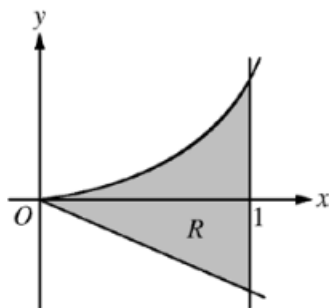
$$f'(x) = \frac{k - 2x}{(x^2 - kx)^2}.$$

- (a) Let  $k = 3$ , so that  $f(x) = \frac{1}{x^2 - 3x}$ . Write an equation for the line tangent to the graph of  $f$  at the point whose  $x$ -coordinate is 4.
- (b) Let  $k = 4$ , so that  $f(x) = \frac{1}{x^2 - 4x}$ . Determine whether  $f$  has a relative minimum, a relative maximum, or neither at  $x = 2$ . Justify your answer.
- (c) Find the value of  $k$  for which  $f$  has a critical point at  $x = -5$ .
- (d) Let  $k = 6$ , so that  $f(x) = \frac{1}{x^2 - 6x}$ . Find the partial fraction decomposition for the function  $f$ .

Find  $\int f(x) dx$ .

Let  $R$  be the shaded region bounded by the graph of  $y = xe^{x^2}$ , the line  $y = -2x$ , and the vertical line  $x = 1$ , as shown in the figure above.

- (a) Find the area of  $R$ .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -2$ .
- (c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of  $R$ .



At time  $t$ , a particle moving in the  $xy$ -plane is at position  $(x(t), y(t))$ , where  $x(t)$  and  $y(t)$  are not explicitly given. For  $t \geq 0$ ,  $\frac{dx}{dt} = 4t + 1$  and  $\frac{dy}{dt} = \sin(t^2)$ . At time  $t = 0$ ,  $x(0) = 0$  and  $y(0) = -4$ .

- (a) Find the speed of the particle at time  $t = 3$ , and find the acceleration vector of the particle at time  $t = 3$ .
- (b) Find the slope of the line tangent to the path of the particle at time  $t = 3$ .
- (c) Find the position of the particle at time  $t = 3$ .
- (d) Find the total distance traveled by the particle over the time interval  $0 \leq t \leq 3$ .

A particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ , where  $x(t) = t^2 - 4t + 8$  and  $y(t)$  is not explicitly given. Both  $x$  and  $y$  are measured in meters, and  $t$  is measured in seconds. It is known that  $\frac{dy}{dt} = te^{t-3} - 1$ .

- (a) Find the speed of the particle at time  $t = 3$  seconds.
- (b) Find the total distance traveled by the particle for  $0 \leq t \leq 4$  seconds.
- (c) Find the time  $t$ ,  $0 \leq t \leq 4$ , when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.
- (d) There is a point with  $x$ -coordinate 5 through which the particle passes twice. Find each of the following.
  - (i) The two values of  $t$  when that occurs
  - (ii) The slopes of the lines tangent to the particle's path at that point
  - (iii) The  $y$ -coordinate of that point, given  $y(2) = 3 + \frac{1}{e}$