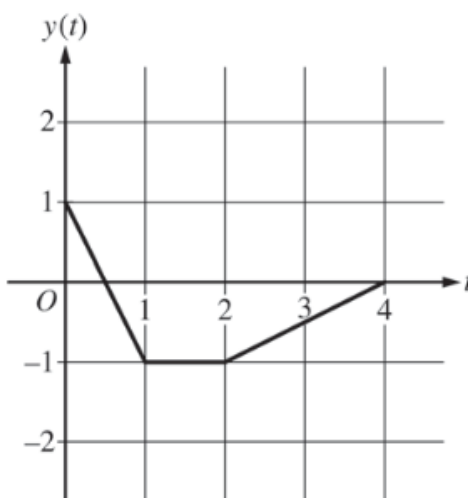




AP Calculus BC – AP Exam Review Project –Free Response Problems → Group 3

Year & #	2016 2	2015 6	2013 1	2012 3	2011 2	
National Average	4.14	3.96	3.87	4.29	5.48	
Type of Problem	Slope, Speed, Distance	Maclaurin & Ratio Test	WP	Max, Min, & Geometric	Trapezoid	Euler Riemann



At time t , the position of a particle moving in the xy -plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y , consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

- Find the position of the particle at $t = 3$.
- Find the slope of the line tangent to the path of the particle at $t = 3$.
- Find the speed of the particle at $t = 3$.
- Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

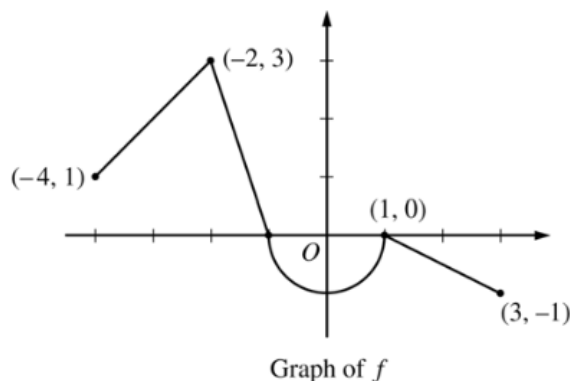
The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n = x - \frac{3}{2}x^2 + 3x^3 - \cdots + \frac{(-3)^{n-1}}{n} x^n + \cdots$ and converges to $f(x)$ for $|x| < R$, where R is the radius of convergence of the Maclaurin series.

- (a) Use the ratio test to find R .
- (b) Write the first four nonzero terms of the Maclaurin series for f' , the derivative of f . Express f' as a rational function for $|x| < R$.
- (c) Write the first four nonzero terms of the Maclaurin series for e^x . Use the Maclaurin series for e^x to write the third-degree Taylor polynomial for $g(x) = e^x f(x)$ about $x = 0$.

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45 \cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \leq t \leq 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \leq t \leq 8$, the plant processes gravel at a constant rate of 100 tons per hour.

- (a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.
- (b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
- (c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.
- (d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t) dt$.



- Find the values of $g(2)$ and $g(-2)$.
- For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.
- Find the x -coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.
- For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?