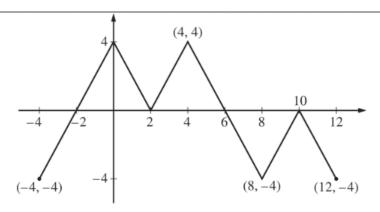


## $\exists$ AP Calculus BC – AP Exam Review Project –Free Response Problems $\Rightarrow$ **Group 2**

Year & #	2016	2014	2013	2012	2010	
	3	1	6	1	4	
National Average	4.15	4.26	5.14	5.88	2.28	
Type of Problem	Max Min Integral Graph - poi	Average Rate	Taylor	Riemann	Area & Volume	Polar WP



Graph of f

The figure above shows the graph of the piecewise-linear function f. For  $-4 \le x \le 12$ , the function g is defined by  $g(x) = \int_2^x f(t) dt$ .

- (a) Does g have a relative minimum, a relative maximum, or neither at x = 10? Justify your answer.
- (b) Does the graph of g have a point of inflection at x = 4? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of g on the interval  $-4 \le x \le 12$ . Justify your answers.
- (d) For  $-4 \le x \le 12$ , find all intervals for which  $g(x) \le 0$ .

Grass clippings are placed in a bin, where they decompose. For  $0 \le t \le 30$ , the amount of grass clippings remaining in the bin is modeled by  $A(t) = 6.687(0.931)^t$ , where A(t) is measured in pounds and t is measured in days.

- (a) Find the average rate of change of A(t) over the interval  $0 \le t \le 30$ . Indicate units of measure.
- (b) Find the value of A'(15). Using correct units, interpret the meaning of the value in the context of the problem.
- (c) Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval 0 ≤ t ≤ 30.
- (d) For t > 30, L(t), the linear approximation to A at t = 30, is a better model for the amount of grass clippings remaining in the bin. Use L(t) to predict the time at which there will be 0.5 pound of grass clippings remaining in the bin. Show the work that leads to your answer.

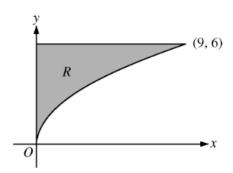
A function f has derivatives of all orders at x = 0. Let  $P_n(x)$  denote the nth-degree Taylor polynomial for f about x = 0.

- (a) It is known that f(0) = -4 and that  $P_1\left(\frac{1}{2}\right) = -3$ . Show that f'(0) = 2.
- (b) It is known that  $f''(0) = -\frac{2}{3}$  and  $f'''(0) = \frac{1}{3}$ . Find  $P_3(x)$ .
- (c) The function h has first derivative given by h'(x) = f(2x). It is known that h(0) = 7. Find the third-degree Taylor polynomial for h about x = 0.

t (minutes)	0	4	9	15	20
W(t) (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time t = 0, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time t = 0. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate W'(12). Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate  $\int_0^{20} W'(t) dt$ . Using correct units, interpret the meaning of  $\int_0^{20} W'(t) dt$  in the context of this problem.
- (c) For  $0 \le t \le 20$ , the average temperature of the water in the tub is  $\frac{1}{20} \int_0^{20} W(t) \, dt$ . Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate  $\frac{1}{20} \int_0^{20} W(t) \, dt$ . Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For  $20 \le t \le 25$ , the function W that models the water temperature has first derivative given by  $W'(t) = 0.4\sqrt{t}\cos(0.06t)$ . Based on the model, what is the temperature of the water at time t = 25?



Let R be the region in the first quadrant bounded by the graph of  $y = 2\sqrt{x}$ , the horizontal line y = 6, and the y-axis, as shown in the figure above.

- (a) Find the area of R.
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line y = 7.
- (c) Region R is the base of a solid. For each y, where  $0 \le y \le 6$ , the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.