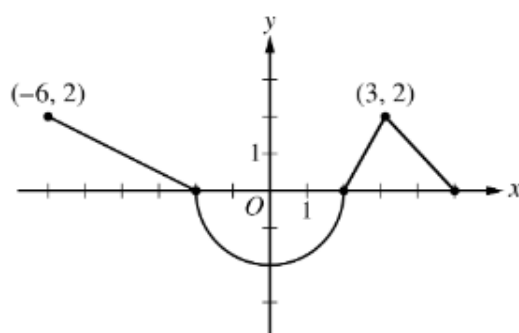




AP Calculus BC – AP Exam Review Project –Free Response Problems → Group 10

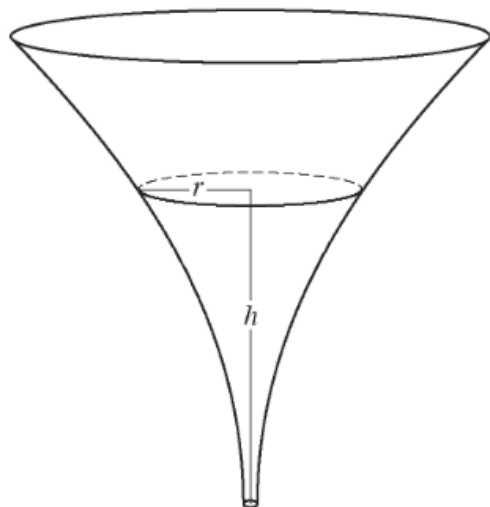
Year & #	2017 3	2016 5	2013 2	2011 5	2008 2	
National Average	5.42	4.19	3.44	4.13	5.10	
Type of Problem	F' , f'' , semi circle graph	Average Value Related Rate	Polar	2^{nd} Derivative	Trapezoid	Polar Logistic Growth



Graph of f'

The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.

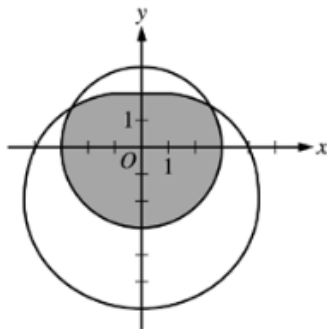
- Find the values of $f(-6)$ and $f(5)$.
- On what intervals is f increasing? Justify your answer.
- Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
- For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.



The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h , the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \leq h \leq 10$. The units of r and h are inches.

- Find the average value of the radius of the funnel.
- Find the volume of the funnel.
- The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

The graphs of the polar curves $r = 3$ and $r = 4 - 2\sin \theta$ are shown in the figure above. The curves intersect when $\theta = \frac{\pi}{6}$ and $\theta = \frac{5\pi}{6}$.



- Let S be the shaded region that is inside the graph of $r = 3$ and also inside the graph of $r = 4 - 2\sin \theta$. Find the area of S .
- A particle moves along the polar curve $r = 4 - 2\sin \theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the interval $1 \leq t \leq 2$ for which the x -coordinate of the particle's position is -1 .
- For the particle described in part (b), find the position vector in terms of t . Find the velocity vector at time $t = 1.5$.

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

- Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).
- Find $\frac{d^2W}{dt^2}$ in terms of W . Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.
- Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.

- Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?