



AP Calculus BC – AP Exam Review Project –Free Response Problems → Group 1

Year & #	2016 1	2014 2	2013 5	2009 3	2009 6	
National Average	5.70	3.96	3.25	1.92	1.79	
Type of Problem	Riemann	Polar	Euler's Method	Total Distance	Maclaurin /Taylor	Continuity Area/Volume

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \leq t \leq 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \leq t \leq 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

- Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.
- Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- For $0 \leq t \leq 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.

- (a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R .

- (b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at

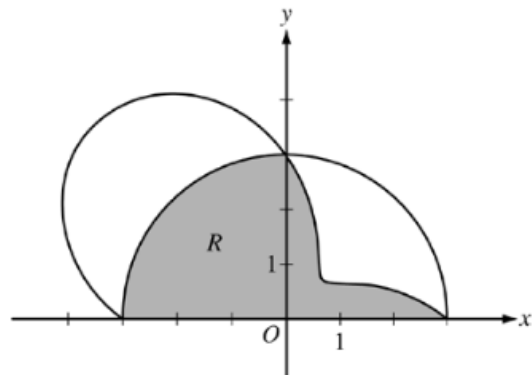
$$\theta = \frac{\pi}{6}.$$

- (c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$.

Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

- (d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value

$$\text{of } \frac{dr}{dt} \text{ at } \theta = \frac{\pi}{6}.$$



Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

- (a) Find $\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

- (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

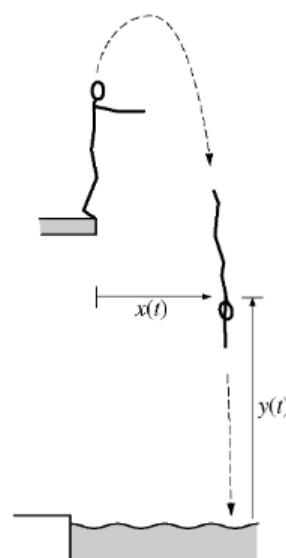
- (c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \quad \text{and} \quad \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \leq t \leq A$, where A is the time that the diver's shoulders enter the water.

- Find the maximum vertical distance from the water surface to the diver's shoulders.
- Find A , the time that the diver's shoulders enter the water.
- Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.
- Find the angle θ , $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.



Note: Figure not drawn to scale.

The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function f is defined

by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function f has derivatives of all orders at $x = 1$.

- Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.
- Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.
- Use the ratio test to find the interval of convergence for the Taylor series found in part (b).
- Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.