



# AP Calculus BC Practice Exam # 2

jlb

- 1.** If  $a$ ,  $b$ , and  $c$  are constants, what is  $\lim_{z \rightarrow a} (bz^2 + c^2 z)$ ?
- A.  $a^2 b + ac^2$   
B.  $az^2 + a^2 z$   
C.  $bz^2 + ac^2$   
D.  $a^2 b + c^2 z$   
E.  $bz^2 + c^2 z$
- 2.** The slope of the line normal to the curve  $g(x) = \sin \frac{x}{2} + 3x$  at the point where  $x = \frac{\pi}{7}$  is approximately
- A. -0.287  
B. 0.449  
C. 1.276  
D. 2.671  
E. 3.487
- 4.** A point of inflection for the graph of  $y = x^3 + 4x^2 - x + 5 \sin x$  has  $x$  coordinate
- A. -4.467  
B. -3.273  
C. -2.066  
D. -1.059  
E. -.519
- 5.** If  $y = x^{x-2}$ , then  $\frac{dy}{dx} \Big|_{x=3} \approx$
- A. 2.358  
B. 3.761  
C. 4.296  
D. 4.553  
E. none of these
- 3.** Given  $\lim_{x \rightarrow 3^+} f(x) = 2$ , which of the following MUST be true?
- A.  $f(3)$  exists  
B.  $f(x)$  is continuous at  $x = 3$   
C.  $f(3) = 2$   
D.  $\lim_{x \rightarrow 3^-} f(x) = 2$   
E. None of these must be true.
- 6.** A right circular cylindrical can having a volume of  $2\pi$  in<sup>3</sup> is to be constructed. Find the radius of the can for which the total surface area is a minimum.  
 $(V = \pi r^2 h, A = 2\pi r^2 + 2\pi rh)$
- A. 1/4  
B. 1/2  
C. 1  
D. 2  
E. 4

7. If  $g(x) = \frac{1}{2}|3-x|$ , then the value of the derivative of  $g(x)$  at  $x=3$  is

- A.  $-\frac{1}{2}$
- B.  $\frac{1}{2}$
- C. 0
- D. 3
- E. nonexistent

8.  $\int \frac{dx}{x^2 - x - 2} =$

- A.  $-\frac{1}{3} \ln \left| \frac{x+1}{x-2} \right| + C$
- B.  $\ln \left| \frac{x-1}{x+2} \right|^3 + C$
- C.  $-\frac{1}{x} - \ln|x| - \frac{x}{2} + C$
- D.  $\frac{1}{3} - \ln \left| \frac{x-1}{x+2} \right| + C$
- E.  $\ln \left| \frac{x-2}{x+1} \right|^3 + C$

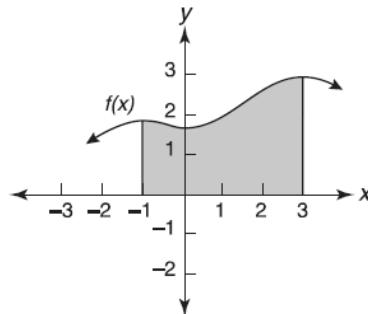
9. For the function  $y = x^{100}$  find  $\frac{d^{100}y}{dx^{100}}$ .

- A. 0
- B. 100
- C.  $(100!)x$
- D. 100!
- E.  $100x$

10.  $\int_1^2 \sin^5 x \, dx \approx$

- A. 0.732
- B. 0.815
- C. 0.867
- D. 0.924
- E. 1.173

11. Which of the following is equal to the shaded area in the figure below?



- A.  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left( \frac{3}{n} \right) f\left(\frac{3i}{n}\right)$
- B.  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left( \frac{4}{n} \right) f\left(-1 + \frac{4i}{n}\right)$
- C.  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left( \frac{3}{n} \right) f\left(-1 + \frac{3i}{n}\right)$
- D.  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left( \frac{4}{n} \right) f\left(\frac{4i}{n}\right)$
- E. none of these

12. What is  $\lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h}$ ?

- A. 0
- B.  $\ln x$
- C.  $x$
- D.  $e^x$
- E. 1

- 13.** The average value of the function  $f(x) = x \sin x$  on the closed interval  $[1, \pi]$  is approximately

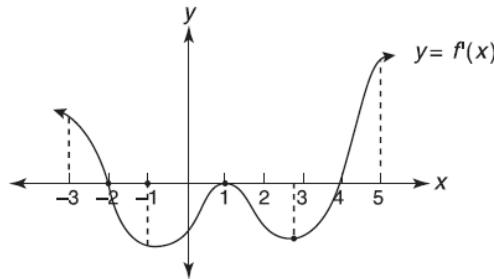
- A. 1.326
- B. 1.467
- C. 2.840
- D. 3.142
- E. 4.076

- 14.** For what values of  $x$  will the series  $\sum_{n=1}^{\infty} \frac{x^{n+1}}{(n+1)^2}$  converge?

- A.  $(-1, 1]$
- B.  $[-1, 1)$
- C.  $(-1, 1)$
- D.  $[-1, 1]$
- E.  $(-\infty, +\infty)$

- 16.** A curve is described by the parametric equations  $x = \frac{1}{4}t^4$  and  $y = \frac{1}{3}t^3$ . The length of this curve from  $t = 0$  to  $t = 2$  is given by

- A.  $\int_0^2 \sqrt{\frac{t^8}{16} + \frac{t^6}{9}} dt$
- B.  $\int_0^2 \sqrt{\frac{t^4}{4} + \frac{t^3}{3}} dt$
- C.  $\int_0^2 \sqrt{\frac{t^{10}}{400} + \frac{t^8}{144}} dt$
- D.  $\int_0^2 \sqrt{(t^3 - t^2)^2} dt$
- E.  $\int_0^2 \sqrt{(t^6 - t^4)} dt$



- 15.** When the value of  $\cos 2$  is approximated by using the fourth-degree Taylor polynomial about  $x = 0$ , the value of  $\cos 2$  is

- A.  $1 + \frac{4}{2} - \frac{16}{24}$
- B.  $1 + \frac{4}{2} + \frac{16}{24}$
- C.  $1 - \frac{4}{2} - \frac{16}{24}$
- D.  $-1 - \frac{4}{2} + \frac{16}{64}$
- E.  $1 - \frac{4}{2} + \frac{16}{24}$

- 17.** Above is the graph of  $f'(x)$ . On what interval (5) is the graph of  $f(x)$  concave upwards?

- A.  $-3 < x < 1$  and  $1 < x < -1$
- B.  $-2 < x < 1$  and  $1 < x < 4$
- C.  $-1 < x < 3$
- D.  $-1 < x < 1$  and  $3 < x < 5$
- E.  $-3 < x < 1$

18. The region bounded by the graphs of  $y = x^2 - 5x + 6$  and  $y = 0$  is rotated about the  $y$ -axis. The volume of the resulting solid is

- A.  $10\pi$
- B.  $52\pi$
- C.  $5\pi/6$
- D.  $5\pi$
- E.  $19\pi/3$

19. Let  $A$  be the region bounded by  $y = \ln x$ , the  $x$ -axis, and the line  $x = e$ . Which of the following represents the volume of the solid generated when  $A$  is revolved around the  $y$ -axis?

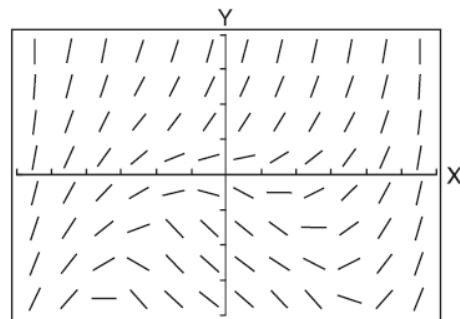
- A.  $\pi \int_0^1 (e^2 - e^{2y}) dy$
- B.  $\pi \int_0^1 (e^2 - e^y) dy$
- C.  $\pi \int_0^1 e^{2y} dy$
- D.  $2\pi \int_1^e (e^2 - e^{2y}) dy$
- E.  $\pi \int_0^1 e^2 dy$

20. Which of the following series diverge?

I.  $\sum_{n=1}^{\infty} \frac{4}{3^{n+1}}$   
 II.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n}$

III.  $\sum_{n=3}^{\infty} \frac{3}{n^2 + 1}$

- A. none
- B. I only
- C. II only
- D. III only
- E. All of them



21. Above is the slope field for which of the following differential equations?

- A.  $\frac{dy}{dx} = \sin x$
- B.  $\frac{dy}{dx} = x^2 + y$
- C.  $\frac{dy}{dx} = 2x + 3$
- D.  $\frac{dy}{dx} = 3y - 2$
- E.  $\frac{dy}{dx} = \cos y$

22. A particle moves in the  $xy$ -plane so that at any time  $t > 0$ ,  $x = \frac{1}{4}t^4 - 3t$  and  $y = \frac{1}{3}(3t - 5)^4$ . The acceleration vector of the particle at  $t = 2$  is

- A.  $(-2, \frac{1}{3})$
- B.  $(5, 4)$
- C.  $(12, 36)$
- D.  $(12, 4)$
- E.  $(\frac{-52}{15}, \frac{1}{810})$

# Free Response – Practice Exam # 2

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1. Let  $y = f(x)$  be a continuous function such that  $\frac{dy}{dx} = 3xy$ ,  $x \geq 0$ , and  $f(0) = 12$ .
- (a) Find  $f(x)$ .  
(b) Find  $f^{-1}(x)$ .
2. Let  $f$  be a function that has derivatives of all orders for all numbers. Assume  $f(1) = 3$ ,  $f'(1) = -2$ ,  $f''(1) = 7$ , and  $f'''(1) = -5$
- (a) Find the third-degree Taylor polynomial for  $f$  about  $x = 3$  and use it to approximate  $f(3.2)$ .  
(b) Write the fourth-degree Taylor polynomial for  $g$ , where  $g(x) = f(x^2 + 3)$  about 3.  
(c) Write the third-degree Taylor polynomial for  $h$ , where  $f(x) = \int_3^x f(t) dt$  about  $x = 3$ .
3. Let  $f$  be the function defined as
- $$f(x) = \begin{cases} 6x + 12 & \text{for } x \leq -2 \\ ax^3 + b & \text{for } -2 < x < 1 \\ 2x + \frac{5}{2} & \text{for } x \geq 1 \end{cases}$$
- (a) Find values for  $a$  and  $b$  such that  $f(x)$  is continuous. Use the definition of continuity to justify your answer.  
(b) For the values you found in part (a), is  $f(x)$  differentiable at  $x = -2$ ? at  $x = 1$ ? Use the definition of the derivative to justify your answer.

